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Detection of coupling with linear and nonlinear synchronization measures for EEG

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Abstract— **There has been extensive research aimed at measuring synchronization to study the relationships between complex time series, such as electroencephalography (EEG). We compare six synchronization measures: the linear measures of cross-correlation, coherence and partial coherence, and three nonlinear similarity measures, namely correntropy, phase index and mutual information. We apply these measures to simulated data (unidirectionally coupled Hénon maps) to test the detection of nonlinear and nonstationary interdependence, including in the presence of noise. We also apply these measures to simulated EEG. The results suggest different measures have both good and bad features. No measure is the clear winner and no method completely fails. "Best measure" depends on the particular data and aim of the research.**

I. Introduction

There has been wide-ranging research aimed at detecting underlying relationships (which may be nonlinear and/or nonstationary) in multi-output dynamic systems, to give useful insight into their spatio-temporal organization [1]. Previous studies of coupled identical and nonidentical systems have focused on a single measure, and there is no thorough comparison of many measures [1-7].

The classical measures use linear approaches, in particular cross-correlation and coherence [8]. Cross-correlation measures the linear correlation between two signals in the time domain, while coherence measures linear relationships between signals in the frequency domain and is calculated as the normalized cross-spectral density of the two signals [9]. Partial coherence measures the same relationship but after (linear) dependence on other signals has been removed [10]. Linear measures have generally been assumed to only capture linear relationships, and so these measures are rarely used to detect nonlinear relationships [11].

Nonlinear measures are often based on phase information, such as phase index, or on information theory, such as correntropy or mutual information [12]. Phase index looks for a consistent difference in instantaneous phase between the two signals [13]. Correntropy extends the correlation function to nonlinear space [5]. Mutual information quantitates how much information is shared between two signals, based on the joint and marginal distributions of the signals. It is noted that accurate estimation of these distributions generally requires a large amount of noise-free stationary data, which limits the practical application of such measures [10].

Electroencephalography (EEG) is the recording of brain electrical activity at the scalp. EEG is fundamentally a nonstationary signal due to the time varying nature of brain activity. The recorded EEG signals are typically examined in the frequency range between 0 and 100 Hz. Most of the signal's energy is distributed between 0.5 and 60 Hz and its amplitude is typically between 2 and 100 μV [14]. Therefore, EEG signals overlap in one or both of amplitude and frequency with many other biological signals and external noises. Therefore a good synchronization measure for EEG should be insensitive to noise, including non-brain signals, as well as be able to detect both linear and nonlinear and nonstationary relationships between signals.

In this paper we consider three classical measures (cross-correlation, coherence and partial coherence) and three nonlinear measures (phase index, correntropy and mutual information). All measures are normalized by definition to the range 0 to 1 (we ignore the sign for cross-correlation), except mutual information, which we normalize by dividing by the average self-information of the two input signals. To compare the synchrony measures, we generated synthetic data where we know the true relationship between the signals. First we used a well-understood nonlinear system (coupled Hénon maps), followed by simulated EEG.

This paper is organized as follows. In section II we apply the measures to Hénon map data, looking for increasing coupling strength, nonstationary coupling and addressing the influence of noise. In section III, we apply the measures to simulated EEG signals. In section IV we summarize the results and present our conclusions.

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II. EXPERIMENTS ON COUPLED HÉNON MAPS

Our simulations generate data from a pair of unidirectionally coupled Hénon maps X and Y given by:

$$x(k+1) = 1.4 + b\,x(k-1) - x^2(k)$$
$$y(k+1) = 1.4 + d\,y(k-1) - [\mu\,x(k) + (1-\mu)y(k)]y(k)$$

We analyze three standard cases, commonly referred to as identical systems $(b = d = 0.3)$ and nonidentical systems $(b = 0.3, d = 0.1)$ and $(b = 0.1, d = 0.3)$ [3]. For both cases we varied the coupling strength from $\mu = 0$ (no coupling) to $\mu = 1$ (complete coupling) [3]. Simulations used 10,000 data points and were repeated 10 times with different random initial conditions, with the calculated synchronization measures averaged over the 10 realizations.

Analysis of the identical system has shown an increase in synchronization as the coupling strength increases, but from $\mu = 0.7$ the system switches to perfect synchronization, ie $x(k) = y(k)$ as the maximum Lyapunov exponent goes negative [3]. The nonidentical systems both show increases in synchronization with coupling strength, without achieving perfect synchronization. However, for $(b = 0.3, d = 0.1)$ the maximum Lyapunov exponent goes negative between $\mu = 0.1$ and $\mu = 0.3$, and for $(b = 0.1, d = 0.3)$, the maximum Lyapunov exponent goes negative between $\mu = 0.4$ and $\mu = 0.6$ [3, 7]. In these ranges, stronger synchronization can be expected.

A. *Detecting nonlinear relationships between signals*

We vary the strength of a relationship between signal by varying the coupling strength μ. Fig. 1 shows the synchrony measures against increasing coupling strength. As expected, all measures detect the perfect synchronization that occurs at approximately $\mu = 0.7$ [3]. For $\mu < 0.7$, ie before the onset of perfect synchronization, the nonlinear measures identify lesser levels of coupling as previously reported [3] and similarly reported for other nonlinear synchronization measures [3, 15-17]. Not previously reported, however, the linear measures also similarly detect the increasing level of coupling and identify the onset of perfect synchronization, performing as well as the nonlinear measures. Mutual information and correntropy calculate the lowest levels of synchrony at no coupling, suggesting they are more accurate at estimating weak levels of synchrony.

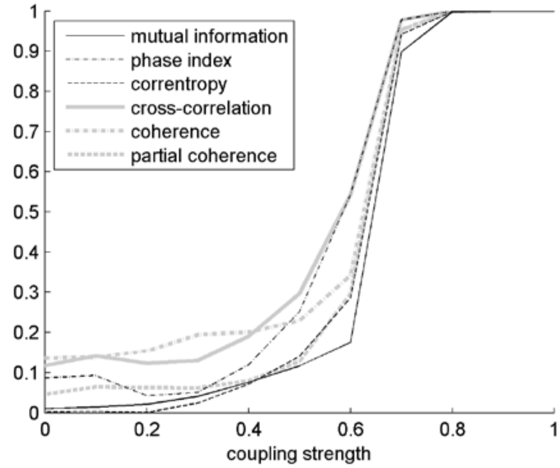

Fig. 1: Synchrony measures applied to coupled Hénon maps (identical systems) with varying coupling strength.

Fig. 2 shows the result for the experiment repeated with nonidentical coupled systems with $(b = 0.3, d = 0.1)$. As reported in [1] both phase index and correntropy measure an increase in synchronization as the coupling strength increases. Again the linear synchronization measures show similar results. The expected increase in synchronization between $\mu = 0.1$ and $\mu = 0.3$ is only visible with the coherence and mutual entropy measures.

Fig. 3 shows the result for the experiment repeated with nonidentical coupled systems with $(b = 0.1, d = 0.3)$. All measures except correntropy show an increase in synchronization with coupling strength, and all but partial coherence show a heightened synchronization between $\mu = 0.4$ and $\mu = 0.6$.

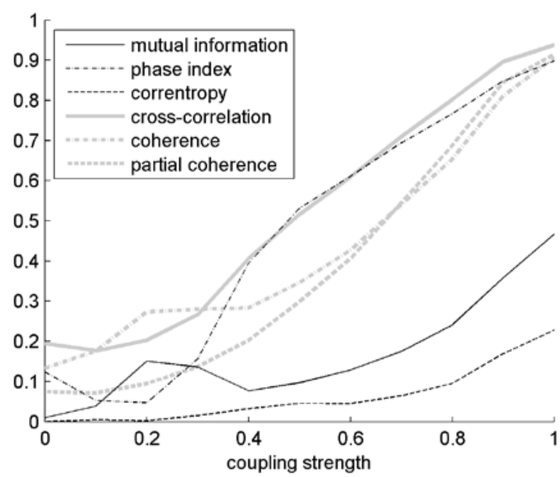

Fig. 2: Synchrony measures applied to coupled Hénon maps (nonidentical systems, $b = 0.3, d = 0.1$) with varying coupling strength.

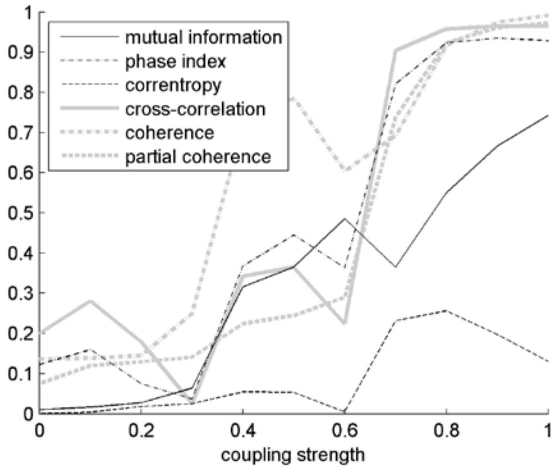

Fig. 3: Synchrony measures applied to coupled Hénon maps (nonidentical systems, $b = 0.1, d = 0.3$) with varying coupling strength.

These experiments demonstrate that with noiseless data the linear synchronization measures generally perform nearly as well as the nonlinear measures in detecting a nonlinear relationship between signals. At very low coupling strengths, mutual information and correntropy generally estimate lower synchrony, outperforming the other measures.

B. Detecting nonstationary relationship between signals

Following the methodology of [11, 18], a model of nonstationarity is to change the coupling strength μ with time. Here we switch the coupling strength of two coupled Hénon maps from $\mu = 0.1$ (weak coupling) to $\mu = 0.9$ (tightly coupling) at $k = 50$ and back to $\mu = 0.1$ at $k = 150$. A sliding window of 50 samples was used to estimate the synchronization measures.

Fig. 4 shows the results for identical systems with the nonlinear measures, and Fig. 5 for the linear measures. For both figures, the change in coupling is indicated by the grey line. All measures show an increase in synchronization starting from $k = 50$ and rising to a maximum after 50 samples, the size of the sliding window. A decrease in synchronization measures is seen from $k = 175$, slightly later than expected. This indicates that it takes approximately 25 samples for the two Hénon maps to commence divergence from their tightly coupled state to their loosely coupled state. All measures do not estimate the loose coupling accurately, due to the limited number of samples used in the estimates. All measures perform equally. Simulations for the nonidentical systems were also run, completely supporting the conclusions above (not shown).

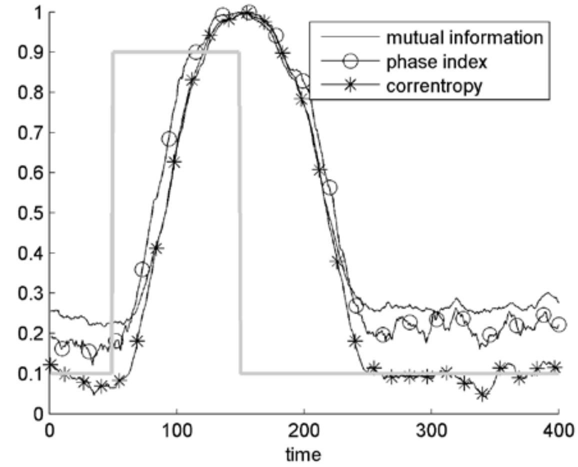

Fig. 4: Nonlinear synchrony measures applied to coupled Hénon maps (identical systems) with nonstationary coupling.

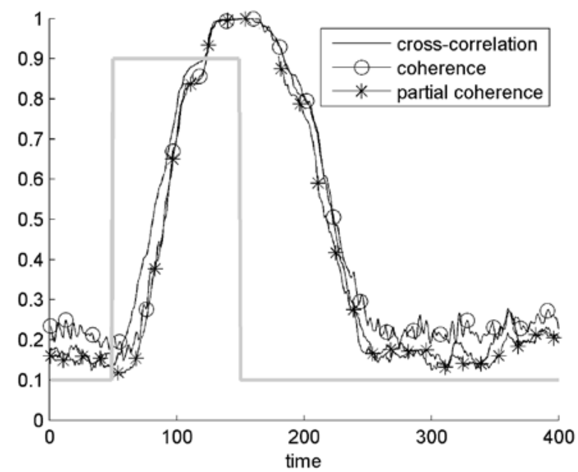

Fig. 5: Linear synchrony measures applied to coupled Hénon maps (identical systems) with nonstationary coupling.

A. Influence of noise

Real signals are contaminated by noise, so a valuable synchronization measure should be robust against noise. Here we consider additive white Gaussian measurement noise at signal-to-noise ratios of 7 dB and 0 dB, levels typically found in EEG.

Fig. 6 shows the performance of the synchronization measures on the identical systems at 7 dB SNR. All measures fail to detect weak coupling below $\mu = 0.3$, and all show a step change at $\mu = 0.7$ when the Hénon maps synchronize. For $0.3 < \mu < 0.7$ as the coupling strengthens, the largest absolute increases are seen in partial coherence, cross-correlation and phase index, and the largest relative increases are seen in phase index, mutual information and partial coherence. Partial coherence obtains the best (largest) estimate when the Hénon maps are synchronized ($\mu > 0.7$).

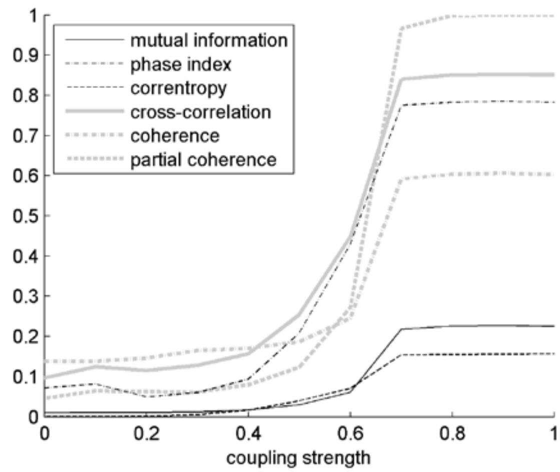

Fig. 6: Synchrony measures applied to coupled Hénon maps (identical systems) with varying coupling strength at 7 dB SNR.

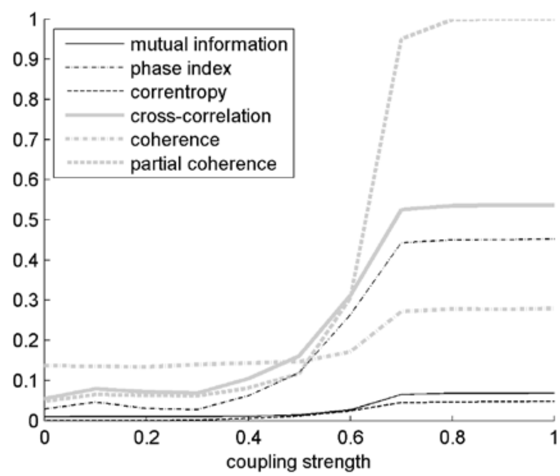

Fig. 7: Synchrony measures applied to coupled Hénon maps (identical systems) with varying coupling strength at 0 dB SNR.

Fig. 7 shows the same experiment at a SNR of 0 dB. The results substantially mimic those in Fig. 6, but with an overall degradation of the estimates due to the extra noise, except for partial coherence. The addition of noise to both nonidentical systems yields results (not shown) that are qualitatively the same as the identical systems and hence reinforce the conclusions above. These experiments suggest phase index and partial coherence are more robust to noise than the other synchrony measures.

III. EXPERIMENTS ON SIMULATED EEG

2 channels of simulated EEG were generated, representing responses to repeated trials. Each trial ran from –1 s to +2 s, and contained a noisy alpha burst with added pink noise at 0 dB. The alpha burst (a Hamming windowed 10 Hz sinusoid) ran from 300 ms to 700 ms with random timing jitter spread uniformly from –5 ms to +5 ms and an additional fixed delay of 10 ms in one channel. The noise added to the burst was pink noise at a SNR of 0 dB.

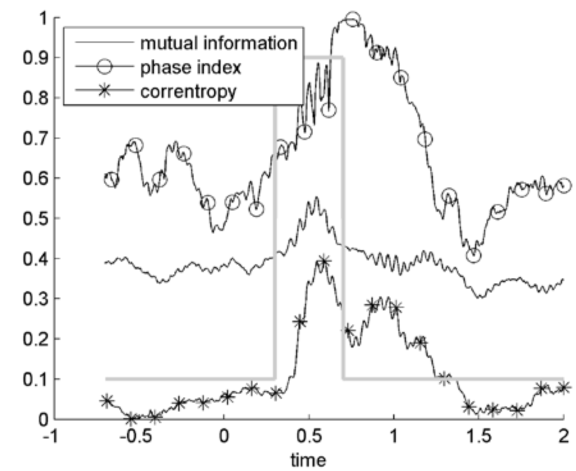

Fig. 8: Nonlinear synchrony measures applied to simulated EEG with varying coupling strength at 0 dB SNR.

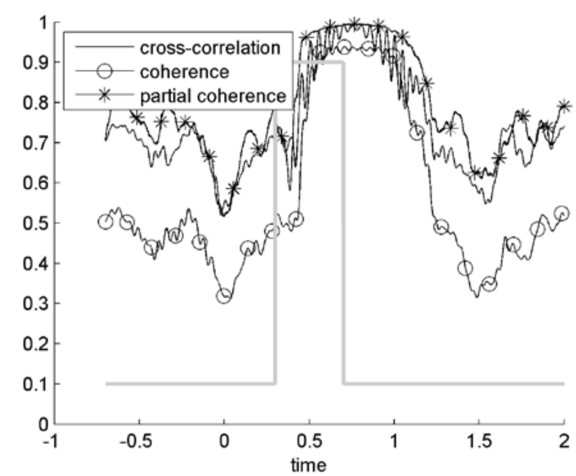

Fig. 9: Linear synchrony measures applied to simulated EEG with varying coupling strength at 0 dB SNR.

The data were analysed for alpha synchrony with six synchrony measures, using a sliding window of 300 ms after preprocessing with a bandpass filter with corner frequencies of 5 Hz and 15 Hz. Fig. 8 shows the synchrony estimates for the nonlinear measures, and Fig. 9 for the linear measures, with both figures indicating the timing of the alpha burst by the grey line. All measures respond as expected to the alpha burst. Coherence shows the largest absolute increase, correntropy shows the largest relative increase.

IV. CONCLUSION

This paper has investigated the performance of six synchronization measures with a view to their use in searching EEG channels for synchronization. EEG is nonlinear and nonstationary. We have demonstrated that linear synchronization measures are able to detect nonlinear relationships and nonstationary relationships. Mutual information and phase index performed well with large block sizes, but were more susceptible to noise at small block sizes. No measure clearly performs better than any other, each measure has its own advantages and disadvantages.

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